frequently used as air speed indicators in aircraft.

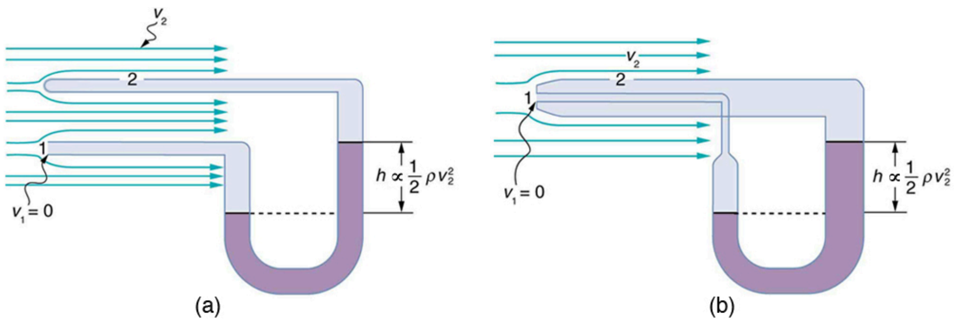

Figure 12.7 Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, and so h is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

12.3 The Most General Applications of Bernoulli's Equation

Torricelli's Theorem

Figure 12.8 shows water gushing from a large tube through a dam. What is its speed as it emerges? Interestingly, if resistance is negligible, the speed is just what it would be if the water fell a distance h from the surface of the reservoir; the water's speed is independent of the size of the opening. Let us check this out. Bernoulli's equation must be used since the depth is not constant. We consider water flowing from the surface (point 1) to the tube's outlet (point 2). Bernoulli's equation as stated in previously is

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2. \tag{12.29}$$

Both P_1 and P_2 equal atmospheric pressure (P_1 is atmospheric pressure because it is the pressure at the top of the reservoir. P_2 must be atmospheric pressure, since the emerging water is surrounded by the atmosphere and cannot have a pressure different from atmospheric pressure.) and subtract out of the equation, leaving

$$\frac{1}{2}\rho v_1^2 + \rho g h_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2. \tag{12.30}$$

Solving this equation for v_2^2, noting that the density ρ cancels (because the fluid is incompressible), yields

$$v_2^2 = v_1^2 + 2g(h_1 - h_2). \tag{12.31}$$

We let $h = h_1 - h_2$; the equation then becomes

$$v_2^2 = v_1^2 + 2gh \tag{12.32}$$

where h is the height dropped by the water. This is simply a kinematic equation for any object falling a distance h with negligible resistance. In fluids, this last equation is called *Torricelli's theorem*. Note that the result is independent of the velocity's direction, just as we found when applying conservation of energy to falling objects.

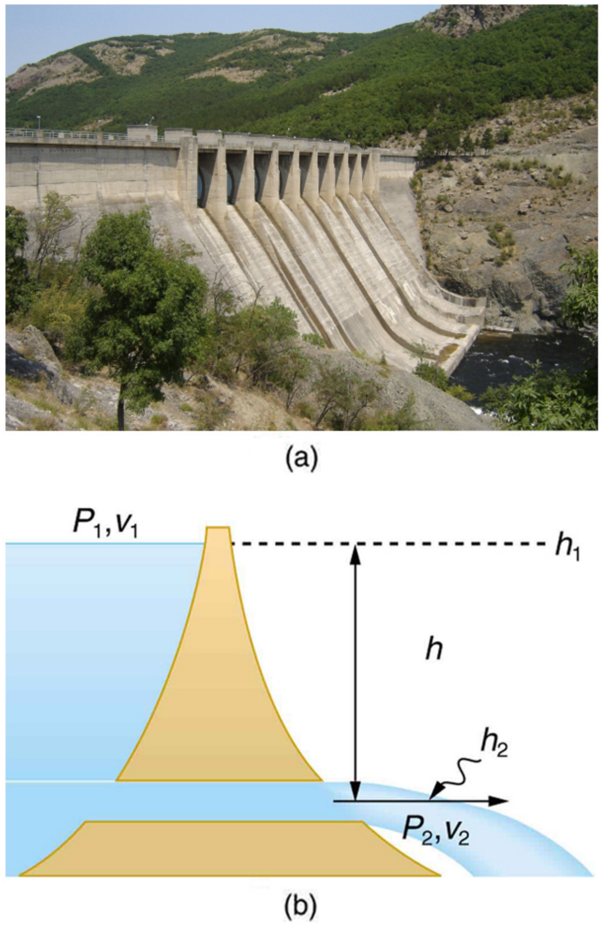

Figure 12.8 (a) Water gushes from the base of the Studen Kladenetz dam in Bulgaria. (credit: Kiril Kapustin; http://www.ImagesFromBulgaria.com) (b) In the absence of significant resistance, water flows from the reservoir with the same speed it would have if it fell the distance h without friction. This is an example of Torricelli's theorem.

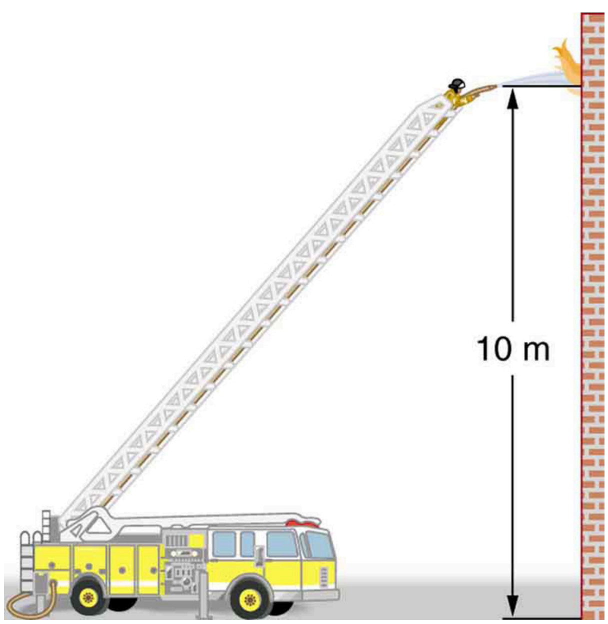

Figure 12.9 Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle, and speed increases in the nozzle. In spite of its lowered pressure, the water can exert a large force on anything it strikes, by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air.

All preceding applications of Bernoulli's equation involved simplifying conditions, such as constant height or constant pressure. The next example is a more general application of Bernoulli's equation in which pressure, velocity, and height all change. (See Figure 12.9.)

EXAMPLE 12.5

Calculating Pressure: A Fire Hose Nozzle

Fire hoses used in major structure fires have inside diameters of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62 \times 10^6\ \text{N/m}^2$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Assuming negligible resistance, what is the pressure in the nozzle?

Strategy

Here we must use Bernoulli's equation to solve for the pressure, since depth is not constant.

Solution

Bernoulli's equation states

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2, \tag{12.33}$$

where the subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds v_1 and v_2. Since $Q = A_1 v_1$, we get

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3}\ \text{m}^3/\text{s}}{\pi(3.20 \times 10^{-2}\ \text{m})^2} = 12.4\ \text{m/s}. \tag{12.34}$$

Similarly, we find

$$v_2 = 56.6\ \text{m/s}. \tag{12.35}$$

(This rather large speed is helpful in reaching the fire.) Now, taking h_1 to be zero, we solve Bernoulli's equation for P_2:

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) - \rho g h_2. \tag{12.36}$$

Substituting known values yields

$$\begin{aligned} P_2 &= \\ &1.62 \times 10^6\ \text{N/m}^2 + \tfrac{1}{2}(1000\ \text{kg/m}^3)\left[(12.4\ \text{m/s})^2 - (56.6\ \text{m/s})^2\right] - (1000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(10.0\ \text{m}) \\ &= 0. \end{aligned} \tag{12.37}$$

Discussion

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus the nozzle pressure equals atmospheric pressure, as it must because the water exits into the atmosphere without changes in its conditions.

Power in Fluid Flow

Power is the *rate* at which work is done or energy in any form is used or supplied. To see the relationship of power to fluid flow, consider Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}. \tag{12.38}$$

All three terms have units of energy per unit volume, as discussed in the previous section. Now, considering units, if we multiply energy per unit volume by flow rate (volume per unit time), we get units of power. That is, $(E/V)(V/t) = E/t$. This means that if we multiply Bernoulli's equation by flow rate Q, we get power. In equation form, this is

$$\left(P + \frac{1}{2}\rho v^2 + \rho g h\right)Q = \text{power}. \tag{12.39}$$

Each term has a clear physical meaning. For example, PQ is the power supplied to a fluid, perhaps by a pump, to give it its pressure P. Similarly, $\frac{1}{2}\rho v^2 Q$ is the power supplied to a fluid to give it its kinetic energy. And $\rho g h Q$ is the power going to gravitational potential energy.

Making Connections: Power

Power is defined as the rate of energy transferred, or E/t. Fluid flow involves several types of power. Each type of power is identified with a specific type of energy being expended or changed in form.

EXAMPLE 12.6

Calculating Power in a Moving Fluid

Suppose the fire hose in the previous example is fed by a pump that receives water through a hose with a 6.40-cm diameter coming from a hydrant with a pressure of $0.700 \times 10^6 \text{ N/m}^2$. What power does the pump supply to the water?

Strategy

Here we must consider energy forms as well as how they relate to fluid flow. Since the input and output hoses have the same diameters and are at the same height, the pump does not change the speed of the water nor its height, and so the water's kinetic energy and gravitational potential energy are unchanged. That means the pump only supplies power to increase water pressure by $0.92 \times 10^6 \text{ N/m}^2$ (from $0.700 \times 10^6 \text{ N/m}^2$ to $1.62 \times 10^6 \text{ N/m}^2$).

Solution

As discussed above, the power associated with pressure is

$$\begin{aligned} \text{power} &= PQ \\ &= \left(0.920 \times 10^6 \text{ N/m}^2\right)\left(40.0 \times 10^{-3} \text{ m}^3/\text{s}\right). \\ &= 3.68 \times 10^4 \text{ W} = 36.8 \text{ kW} \end{aligned} \tag{12.40}$$

Discussion

Such a substantial amount of power requires a large pump, such as is found on some fire trucks. (This kilowatt value converts to about 50 hp.) The pump in this example increases only the water's pressure. If a pump—such as the heart—directly increases velocity and height as well as pressure, we would have to calculate all three terms to find the power it supplies.

12.4 Viscosity and Laminar Flow; Poiseuille's Law

Laminar Flow and Viscosity

When you pour yourself a glass of juice, the liquid flows freely and quickly. But when you pour syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas syrup has high viscosity. In the previous sections we have considered ideal fluids with little or no viscosity. In this section, we will investigate what factors, including viscosity, affect the rate of fluid flow.

The precise definition of viscosity is based on *laminar*, or nonturbulent, flow. Before we can define viscosity, then, we need to define laminar flow and turbulent flow. Figure 12.10 shows both types of flow. **Laminar** flow is characterized by the smooth flow of the fluid in layers that do not mix. Turbulent flow, or **turbulence**, is characterized by eddies and swirls that mix layers of fluid together.